Quantum Mechanics in 1D

1. A matter wave called the wavefunction Ψ can be associated with any particle.
2. The wavefunction Ψ contains all the information that can be known about the particle,

These 2 statements bring up two important questions:

1. How do we obtain this wavefunction Ψ for a given system?
2. How do we extract information from this wavefunction?

Let's look at the answer to the 2nd question first.

Born Interpretation of Ψ

The probability that a particle will be found in the infinitesimal interval dx about the point x, denoted by $P(x)dx$ is given by:

$$\boxed{P(x)dx = |\Psi(\mathrm{x,t})|^2 dx}$$

$P(x) = |\Psi(\mathrm{x,t})|^2$ (probability density function)

$P(x) = probability\ per\ unit\ length\ (in\ 1D)$

A. It follow from this definition that you cannot specify with certainty the location of a particle! You can only specify the probability!

B. $\Psi(\mathrm{x,t})$ itself is NOT a measurable quantity but $|\Psi(\mathrm{x,t})|^2$ is measurable and equal to the probability per unit length (probability density) of finding the particle in the interval dx about the point x.

C. $P(x) = |\Psi(\mathrm{x,t})|^2 = \Psi\Psi^*$ where Ψ^* is the complex conjugate of Ψ.

D. $\Psi(\mathrm{x,t})$ must be single-valued and continuous function of x and t.

E. Because the particle must be somewhere along the x-axis, the sum of the probabilities over all values of x must add up to 1:

$$\boxed{\int_{-\infty}^{+\infty} P(x)dx = \int_{-\infty}^{+\infty} |\Psi(\mathrm{x,t})|^2 dx = 1}$$ Normalization Condition

F. The probability of finding the particle in any finite interval $a \leq x \leq b$ is given by:

$$P_{ab} = \int_a^b |\Psi(\mathrm{x,t})|^2 dx$$

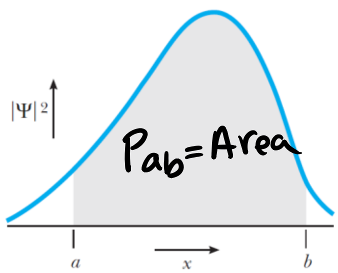

Figure 6.1 The probability for a particle to be in the interval $a \leq x \leq b$ is the area under the curve from a to b of the probability density function $|\Psi(x, t)|^2$.

G. $|\Psi(\mathrm{x,t})|^2 \rightarrow 0$ fast enough as x→ $\pm\infty$ so that the normalization condition holds valid.