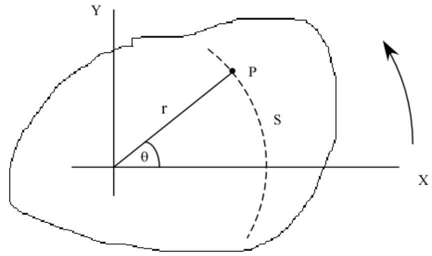
ROTATIONAL MOTION

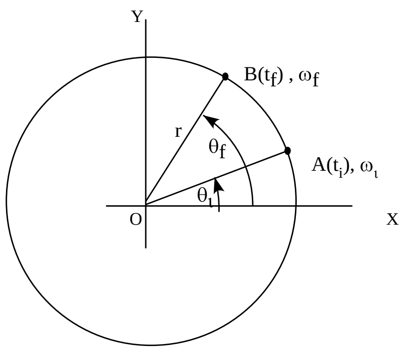
Consider a rigid body rotating CCW about a fixed axis of rotation. Let's look at the motion of a particle located at point P a distance 'r' from the axis of rotation.



As the body rotates the particle moves in a circular path of radius 'r'. The distance the particle moves along this path is related to its angular position 'θ' by the equation:

$$s = r\,\theta$$

Now consider the motion of the particle between two points A and B.



$$\Delta\theta = \theta_f - \theta_i$$ Angular Displacement

$$\omega_{ave} = \frac{\theta_f - \theta_i}{t_f - t_i} = \frac{\Delta\theta}{\Delta t}$$ Average Angular Velocity

$$\omega = \lim_{\Delta t \to 0} \frac{\Delta\theta}{\Delta t} = \frac{d\theta}{dt}$$ Instantaneous Angular Velocity

If the angular velocity changes from ω_i to ω_f in a time $\Delta t = t_f - t_i$, then the particle experiences an angular acceleration:

$$\alpha_{ave} = \frac{\omega_f - \omega_i}{t_f - t_i} = \frac{\Delta\omega}{\Delta t}$$ Average Angular Acceleration

$$\alpha = \lim_{x \to 0} \frac{\Delta\omega}{\Delta t} = \frac{d\omega}{dt}$$ Instantaneous Angular Acceleration

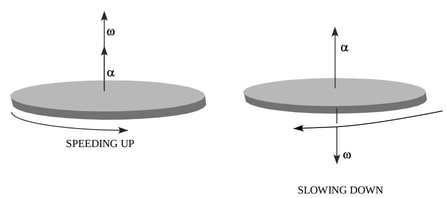
For a body rotating about a fixed axis, every particle on the body has the same rotational quantities $\Delta\theta$, ω, and α. That is $\Delta\theta$, ω, and α describe the rotational motion of the entire body.

Units

[θ] = radians
[ω] = rad/s = s^{-1}
[α] = rad/s^2 = s^{-2}

- ω and α are vector quantities (θ is not a vector because it fails to satisfy the laws of vector addition)
- The direction of $\vec{\omega}$ is given by the Right-Hand Rule (RHR) and the direction of $\vec{\alpha}$ follows from its definition $\boxed{\vec{\alpha} = \frac{d\vec{\omega}}{dt}}$

RHR – Wrap your four right-hand fingers in the direction of rotation. Your extended thumb points in the direction of $\vec{\omega}$.



Mathematically, we have defined the rotational quantities θ, ω, and α similar to how we defined the linear quantities x, v, and a for linear motion. Therefore, the rotational equations of motion with constant angular acceleration, should also be similar.

Linear Motion	Rotational Motion
x	θ
v	ω
t	α
$x = x_o + v_o t + \frac{1}{2}at^2$	$\theta = \theta_o + \omega_o t + \frac{1}{2}\alpha t^2$
$v = v_o + at$	$\omega = \omega_o + \alpha t$
$v^2 = v_o^2 + 2a(x - x_o)$	$\omega^2 = \omega_o^2 + 2\alpha(\theta - \theta_o)$
$x = x_o + \left(\frac{v_o + v}{2}\right)t$	$\theta = \theta_o + \left(\frac{\omega_o + \omega}{2}\right)t$